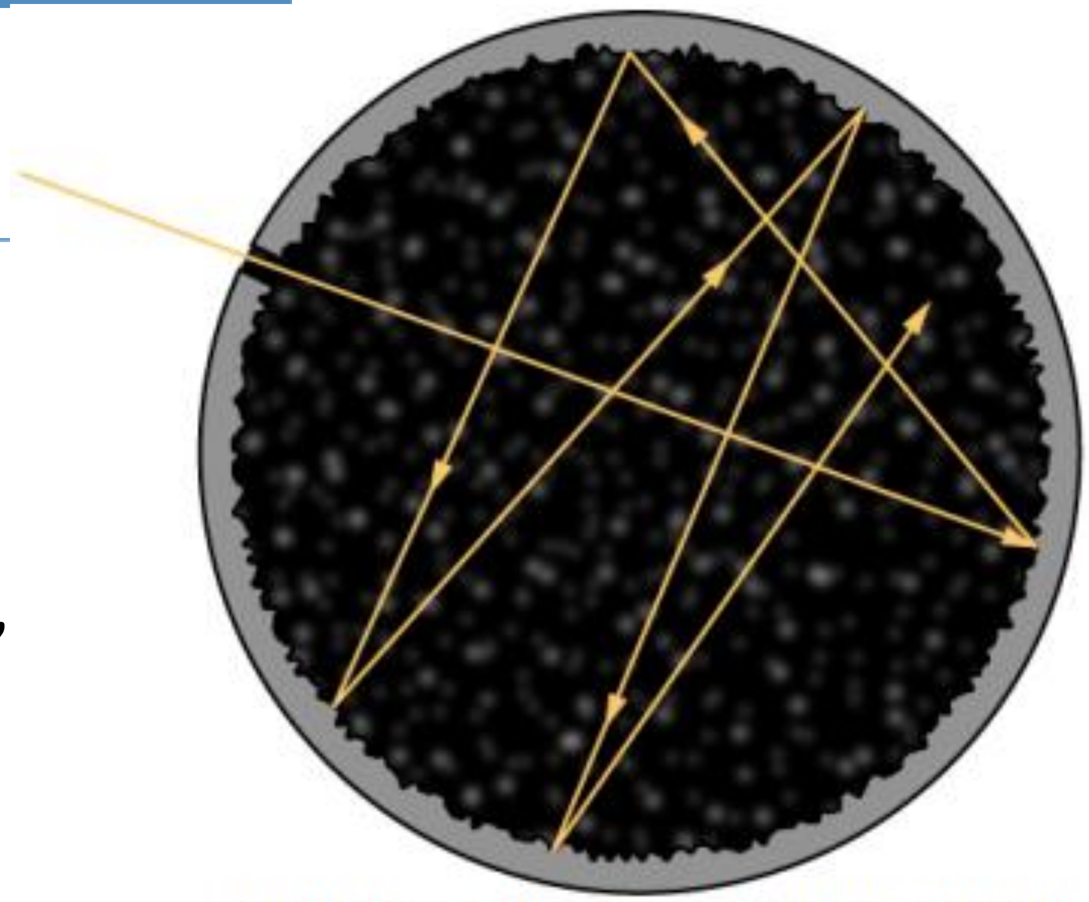
| Blackbody Radiation

Learning Objectives

By the end of this section you will be able to:

- Apply Wien's and Stefan's laws to analyze radiation emitted by a blackbody
- Explain Planck's hypothesis of energy quanta

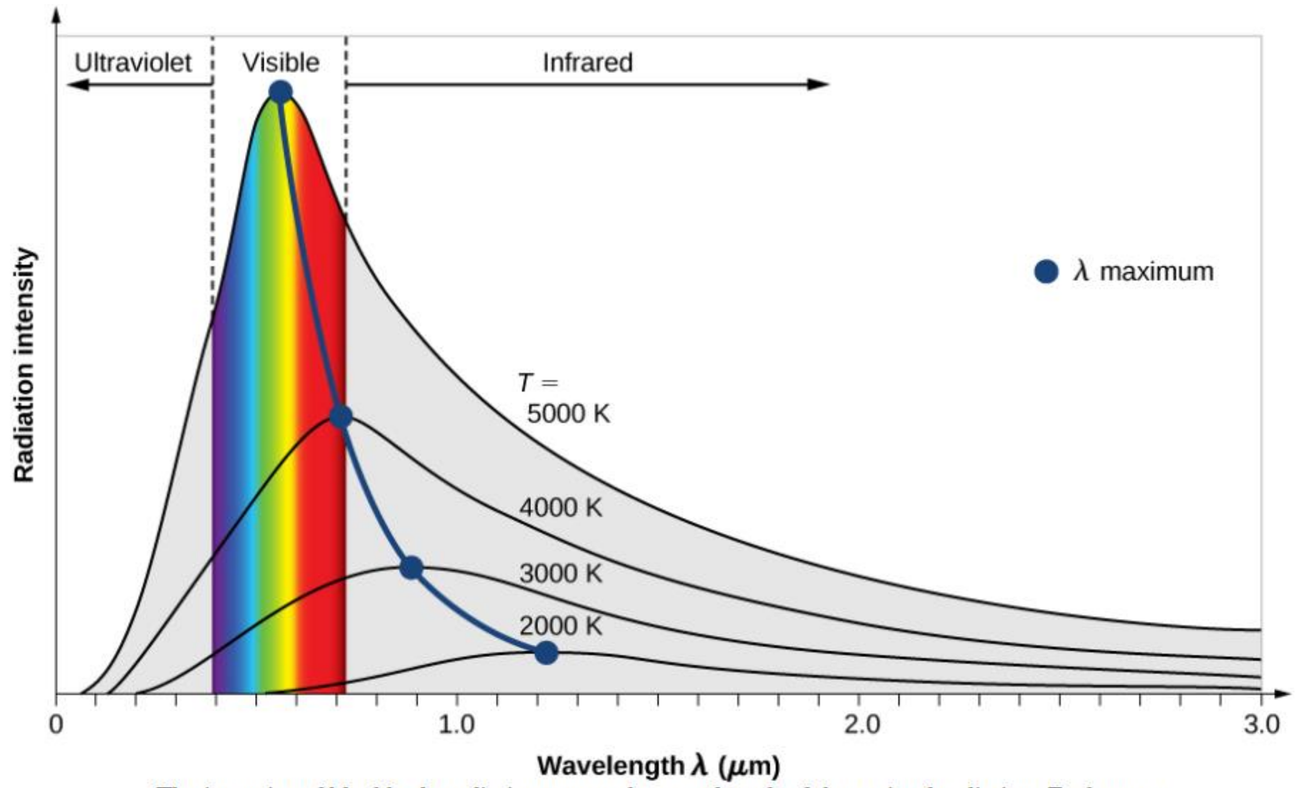
We know by observation that when a body is heated and its temperature rises, the perceived wavelength of its emitted radiation changes from infrared to red, and then from red to orange, and so forth. As its temperature rises, the body glows with the colors corresponding to ever-smaller wavelengths of the electromagnetic spectrum.



A blackbody is physically realized by a small hole in the wall of a cavity radiator.

The temperature (*T*) of the object that emits radiation, or the emitter, determines the wavelength at which the radiated energy is at its maximum. For example, the Sun, whose surface temperature is in the range between 5000 K and 6000 K, radiates most strongly in a range of wavelengths about 560 nm in the visible part of the electromagnetic spectrum. Your body, when at its normal temperature of about 300 K, radiates most strongly in the infrared part of the spectrum. Radiation that is incident on an object is partially absorbed and partially reflected. At thermodynamic equilibrium, the rate at which an object absorbs radiation is the same as the rate at which it emits it. A perfect absorber absorbs all electromagnetic radiation incident on it; such an object is called a blackbody. Electromagnetic waves emitted by a blackbody are called blackbody radiation.

The intensity $I(\lambda, T)$ of blackbody radiation depends on the wavelength λ of the emitted radiation and on the temperature T of the blackbody (Figure 6.3). The function $I(\lambda, T)$ is the power intensity that is radiated per unit wavelength; in other words, it is the power radiated per unit area of the hole in a cavity radiator per unit wavelength. According to this definition, $I(\lambda, T)d\lambda$ is the power per unit area that is emitted in the wavelength interval from λ to $\lambda + d\lambda$.. *Wien's displacement law* is illustrated in the Figure by the curve connecting the maxima on the intensity curves. In these



The intensity of blackbody radiation versus the wavelength of the emitted radiation. Each curve corresponds to a different blackbody temperature, starting with a low temperature (the lowest curve) to a high temperature (the highest curve).

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

where λ_{max} is the position of the maximum in the radiation curve. In other words, λmax is the wavelength at which a blackbody radiates most strongly at a given temperature T On a clear evening during the winter months, if you happen to be in the Northern Hemisphere and look up at the sky, you can see the constellation Orion (The Hunter). One star in this constellation, Rigel, flickers in a blue color and another star, Betelgeuse, has a reddish color,. Which of these two stars is cooler, Betelgeuse or Rigel?

Solution

Writing Wien's law for the blue star and for the red star, we have

$$\lambda_{max}^{(red)} T_{(red)} = 2.898 \times 10^{-3} \text{m} \cdot \text{K} = \lambda_{max}^{(blue)} T_{(blue)}$$

When simplified,

$$T_{(red)} = \frac{\lambda_{max}^{(blue)}}{\lambda_{max}^{(red)}} T_{(blue)} < T_{(blue)}$$

Classical physics
fails to explain the mechanism of blackbody radiation

The blackbody radiation problem was solved in 1900 by Max Planck. Planck innovative idea that Planck introduced in his model : Planck's hypothesis of discrete energy values, which he called *quanta*, assumes that the oscillators inside the cavity walls have quantized energies. This was a brand new idea that went beyond the classical physics (the energy of an oscillator can take on any continuous value). Planck assumed that the energy of an oscillator (*En*) can have only discrete, or quantized, values:

$$E_n = nhf, \text{ where } n = 1, 2, 3, \ldots$$

f is the frequency of Planck's oscillator. The natural number n that enumerates these discrete energies is called a quantum number. The physical constant h is called Planck's constant:

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s} = 4.136 \times 10^{-15}\ \mathrm{eV \cdot s}$$

Planck's Quantum Hypothesis

Planck's hypothesis of energy quanta states that the amount of energy emitted by the oscillator is carried by the quantum of radiation, ΔE :

$$\Delta E = h f$$

Planck’s hypothesis gives the following theoretical expression for the power intensity of emitted radiation per unit wavelength

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

where c is the speed of light in vacuum and kB is Boltzmann’s constant, kB =1.380 × 10^{-23} J/K. The theoretical formula expressed is called Planck’s blackbody radiation law.

Example

Planck’s Quantum Oscillator A quantum oscillator in the cavity wall is vibrating at a frequency of 5.0 × 1014 Hz. Calculate the spacing between its energy levels.

We can substitute the given frequency and Planck’s constant directly into the equation:

$$\Delta E = E_{n+1} - E_n = (n+1)hf - nhf = hf = (6.626 \times 10^{-34}\,\mathrm{J \cdot s})(5.0 \times 10^{14}\,\mathrm{Hz}) = 3.3 \times 10^{-19}\,\mathrm{J}$$

A 1.0-kg mass oscillates at the end of a spring with a spring constant of 1000 N/m. The amplitude of these oscillations is 0.10 m. Use the concept of quantization to find the energy spacing for this classical oscillator. Is the energy quantization significant for macroscopic systems, such as this oscillator?

For the spring constant, $k = 1.0 \times 10^3$ N/m, the frequency f of the mass, $m = 1.0$kg, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.0 \times 10^3 \text{ N/m}}{1.0\text{kg}}} \simeq 5.0 \text{ Hz}$$

The energy quantum that corresponds to this frequency is

$$\Delta E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(5.0\text{Hz}) = 3.3 \times 10^{-33} \text{ J}$$

When vibrations have amplitude $A = 0.10$m, the energy of oscillations is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(1000\text{N/m})(0.1\text{m})^2 = 5.0\text{J}$$